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Multipole Expansion of Bremsstrahlung in Intermediate Energy Heavy Ion Collisions

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Abstract

Using a multipole expansion of the radiated field generated by a classical electric current, we present a way to interprete the bremsstrahlung spectra of low energy heavy ion collisions. We perform the calculation explicitely for the system ^{12}C+^{12}C at 84AMeV and compare the result with the experimental data of E. Grosse et al. Using simple model assumptions for the electromagnetic source current we are able to describe the measured data in terms of coherent photon emission. In this context, the information contained in the measured data is discussed.

1 Introduction

The angular distribution of bremsstrahlung emitted from accelerated charges strongly depends upon the structure of the source current. Correlations between the dynamical properties of the charges show up in the spectrum of radiated photons. A multipole expansion of the radiated field [1, 2, 3, 4] has the advantage, that special structures like symmetries of the current enter directly in the coefficients of the expansion. Dominant structures of the current give dominant terms in the expansion, structures and shapes of the spectrum provide therefore information about the current. At least one can conclude from structures in the spectrum, that the current is somehow structured as well. Using the multipole expansion and simple model assumptions for the electromagnetic source current as well as for the geometry and the dynamics of a heavy ion collision, it is the aim of this paper to show that the experimental data of E. Grosse et al. [7] can be understood as the coherent radiation from a correlated motion of the nucleons. In the following we show how the expansion is obtained. In section 2 we discuss the current that will be used for the calculations. With this current the exact and the approximated spectrum are calculated and compared in section 3. Finally, in section 4 further assumptions, needed to describe a heavy ion collision, are made and included in the calculation, which is explicitely performed for the system ^{12}C+^{12}C at 84AMeV.

Starting from Maxwell's equations, assuming a periodic time dependence of the sources, and looking for a solution of Kirchhoff's differential equation in spherical coordinates one finds a representation of the electric and the magnetic field in terms of Bessel functions and vector spherical harmonics. Due to the duality of $\vec{E}$ and $\vec{B}$ in Maxwell's equations one gets two different solutions for $\vec{E}$ and $\vec{B}$ whose linear combinations are finally the general expressions for the fields.

More generally one can expand the vector potential, i.e. the wave function of the photon, in two transversal and one longitudinal vector field, spanning a 3-dimensional space at each point. The vector fields are designed in a way that they represent outgoing waves in the far field. Since a real photon has only two polarizations one only needs the two transversal fields in the free space outside the sources.

Thus, the magnetic field reads [1]:

$$\vec{B}(\vec{r},\omega) = k \sum_{lm} \left(a_m^{A(l)*}(\omega) \vec{O}_m^{V(l)}(k,\vec{r}) + a_m^{V(l)*}(\omega) \vec{O}_m^{A(l)}(k,\vec{r}) \right) \tag{1}$$

$(k = |\vec{k}|)$,whereas in the far field one gets for the electric field

$$\vec{E}(\vec{r},\omega) = \vec{B} \times \vec{e}(\Omega) \quad . \tag{2}$$

The functions $\vec{O}_m^{V,A(l)}(k,\vec{r})$ are defined by

$$\begin{aligned} \vec{O}_m^{A(l)}(k,\vec{r}) &\stackrel{def}{=} \frac{O_l(k,r)}{r} i^l \vec{Y}_m^{(l,1)l}(\Omega) \\ \vec{O}_m^{V(l)}(k,\vec{r}) &\stackrel{def}{=} \frac{1}{k} \vec{\nabla} \times \vec{O}_m^{A(l)}(k,\vec{r}) \end{aligned} \tag{3}$$

with O_l the outgoing spherical Bessel function. The coefficients of the expansion of $\vec{B}$ in terms of the $\vec{O}_m^{V,A(l)}$ read [2]:

$$a_m^{V,A(l)}(\omega) = \frac{4\pi}{k} \int d^3\vec{r} \vec{j}(\vec{r},\omega) \vec{F}_m^{V,A(l)}(k,\vec{r}) \tag{4}$$

and obey the relation

$$a_m^{V,A(l)*}(\omega) = (-)^{l+m} a_{-m}^{V,A(l)}(-\omega). \tag{5}$$

The $\vec{F}_m^{V,A(l)}(k,\vec{r})$ are defined in the same way as the $\vec{O}_m^{V,A(l)}$ above with the regular spherical Bessel function

$$F_l(kr) = \frac{kr}{2i^l} \int_{-1}^{1} e^{ikr\cos\theta} P_l(\cos\theta) d\cos\theta \quad , \tag{6}$$

$P_l(\cos\theta)$ being the Legendre polynomials. The expressions for the coefficients can be simplified by rewriting the current $\vec{j}(\vec{r},\omega)$ in terms of its Fourier transform and using Rayleigh's plane wave expansion for the exponential function

$$\begin{aligned} \vec{j}(\vec{r},\omega) &= \frac{1}{(2\pi)^3} \int d^3\vec{k}\, e^{i\vec{k}\vec{r}}\, \vec{j}(\vec{k},\omega) \\ &= \frac{2}{(2\pi)^2} \int k^2 dk \sum_{lm} i^{2l} \frac{F_l(kr)}{kr} Y_m^{(l)*}(\Omega_r) \vec{j}_m^{(l)}(\omega) \end{aligned} \tag{7}$$

with

$$\vec{j}(\vec{k},\omega) = \sum_{lm} i^l Y_m^{(l)*}(\Omega_k)\vec{j}_m^{(l)}(\omega)\ ,\ \ \vec{j}_m^{(l)}(\omega) = \int d\Omega_k i^{-l} Y_m^{(l)}(\Omega_k)\vec{j}(\vec{k},\omega)\ \ . \tag{8}$$

Further one uses $\vec{Y}_m^{(l,1)j}(\Omega) \equiv \sum_{m'm''} Y_{m'}^{(l)}(\Omega)\vec{e}_{m''}^{\,(1)} \left(\begin{array}{cc|c} l & 1 & j \\ m' & m'' & m \end{array} \right)$; $\vec{e}_{0}^{\,(1)} = i\vec{e}_z$, $\vec{e}_{\pm 1}^{\,(1)} = \frac{\mp i}{\sqrt{2}}(\vec{e}_x \pm i\vec{e}_y)$ and finally gets

$$\begin{aligned} a_m^{V(l)}(\omega) &= (-)^l \Bigg[i^{(l-1)}\sqrt{\tfrac{l+1}{2l+1}} \sum_{m'm''} \left(\begin{array}{cc|c} l-1 & 1 & l \\ m' & m'' & m \end{array} \right) \vec{e}_{m''}^{\,(1)} \cdot \vec{j}_{m'}^{(l-1)}(\omega) \\ &\quad + i^{(l+1)}\sqrt{\tfrac{l}{2l+1}} \sum_{m'm''} \left(\begin{array}{cc|c} l+1 & 1 & l \\ m' & m'' & m \end{array} \right) \vec{e}_{m''}^{\,(1)} \cdot \vec{j}_{m'}^{(l+1)}(\omega) \Bigg] \end{aligned} \tag{9}$$

$$a_m^{A(l)}(\omega) = (-)^l i^l \sum_{m'm''} \left(\begin{array}{cc|c} l & 1 & l \\ m' & m'' & m \end{array} \right) \vec{e}_{m''}^{\,(1)} \cdot \vec{j}_{m'}^{(l)}(\omega)\ \ .$$

Throughout the calculation we used the orthogonality relations

$$\int_0^\infty dr F_l(kr)F_l(k'r) = \frac{\pi}{2}\delta(k-k')$$

$$\int d\Omega Y_m^{(l)*}(\Omega)Y_{m'}^{(l')}(\Omega) = \delta_{ll'}\delta_{mm'}\ \ .$$

The complex conjugate of $\vec{j}_m^{(l)}(\omega)$ reads $\vec{j}_m^{(l)*}(\omega) = (-)^m \vec{j}_{-m}^{(l)}(-\omega)$.

2 The current

The classical 4-current of a charged particle moving on a trajectory $\vec{x}(\tau)$ in proper time can be written as [5]

$$j^\mu(\vec{x},t) = e\frac{dx^\mu}{dt}\delta^3[\vec{x}-\vec{x}(\tau)]\ |_{t=x^0(\tau)} = e\int d\tau \frac{dx^\mu}{d\tau}\delta^4[x-x(\tau)]\ \ . \tag{10}$$

Rewriting the δ-function by its Fourier integral one finds, that the current for a colliding particle can be understood as the Fourier transform of

$$j^\mu(k) = -ie\left(\frac{p_i^\mu}{k\cdot p_i} - \frac{p_f^\mu}{k\cdot p_f}\right) \tag{11}$$

assuming that the collision can be described by a simple kink in the trajectory. Coulomb deflection effects, which are important in low-energy collisions, are neglected here. Generalizing (11) to the case of several particles suffering several collisions we obtain

$$j^\mu(k) = -ie\sum_i\sum_j\left(\frac{p_{i-1j}^\mu}{k\cdot p_{i-1j}} - \frac{p_{ij}^\mu}{k\cdot p_{ij}}\right)e^{ik\cdot x_{ij}}\ \ . \tag{12}$$

where the indices count vertices (i) and particles (j), respectively. x_{ij} in the equation above accounts for the space-time history of the scenario.

In the following we will make a specific model for the current, considering four colliding particles with equal charge and mass. These particles undergo two subsequent and independent two-body collisions. The first pair interacts at the space time point $x^\mu = (0, \vec{(0)})$ and the remaining two collide at $y^\mu = (y_0, \vec{y})$. For simplicity both collisions are assumed to share the same reaction plane and absolute value of the deflection angle. The Fourier transform of the current in the center of mass frame then reads:

$$\begin{aligned}
j_0(\vec{k},\omega) &= -\frac{ie}{\omega}\Bigg(T(-a,b)+T(a,-b)-\frac{1}{1+\beta_i x}-\frac{1}{1-\beta_i x} \\
&\quad +e^{iky}\left(T(-a,-b)+T(a,b)-\frac{1}{1+\beta_i x}-\frac{1}{1-\beta_i x}\right)\Bigg) \\
j_\perp(\vec{k},\omega) &= \frac{ie}{\omega}b\left(T(-a,b)-T(a,-b)+e^{iky}\left(-T(-a,-b)+T(a,b)\right)\right) \qquad (13) \\
j_\parallel(\vec{k},\omega) &= \frac{ie}{\omega}\Bigg(a\left(T(-a,b)-T(a,-b)\right)-\beta_i\left(\frac{1}{1+\beta_i x}-\frac{1}{1-\beta_i x}\right) \\
&\quad +e^{iky}\left(a\left(T(-a,-b)-T(a,b)\right)-\beta_i\left(\frac{1}{1+\beta_i x}-\frac{1}{1-\beta_i x}\right)\right)\Bigg)
\end{aligned}$$

with

$$T(a,b) = \frac{1}{1-ax+b\sqrt{1-x^2}\cos\varphi_k}$$

$a = \beta_\parallel$, $b = \beta_\perp$ and $x = \cos\theta_k$. For sake of simplicity we only consider a temporal distance $\Delta t = y_0$ between the second collision and the first collision. The current (13) has therefore been constructed in a symmetric manner. For $\Delta t = 0$ it is not only invariant under a parity transformation but also mirror symmetric with respect to axes both parallel and transverse to the beam direction. These symmetries are broken with finite Δt.

3 The spectrum

The spectrum of bremsstrahlung photons radiated by charged particles is given by the simple expression [5]

$$\frac{dN^\gamma}{d\tilde{k}} = -\mid j^\mu(k)\mid^2 \qquad (14)$$

with $d\tilde{k} = \frac{1}{(2\pi)^3}\frac{d^3k}{2k^0}$. The angular dependence of the spectrum is determined by the scalar products in the denominators of (12) $p^\mu k_\mu = p_0 k_0 - |\vec{p}||\vec{k}|\vec{e}(\Omega_p)\cdot\vec{e}(\Omega_k)$, $\vec{e}(\Omega_p)\cdot\vec{e}(\Omega_k) = \cos\theta_p\cos\theta_k + \sin\theta_p\sin\theta_k\cos(\varphi_k-\varphi_p)$.

The spectrum of radiated energy of the process (13) therefore reads ($I = \omega N$)

$$\frac{dI}{d\omega d\Omega_k} = -\frac{\omega^2}{2(2\pi)^3} \mid j^\mu(k) \mid^2 \quad . \tag{15}$$

Inserting the current (13) and averaging over the azimuthal angle φ_k, leads to

$$\begin{aligned} \frac{dI}{d\omega d\Omega_k} &= -\frac{\alpha}{2\pi^2}\Bigg[\frac{8\cos^2(\frac{\omega\Delta t}{2})}{(1-\beta_i^2x^2)}\left(\frac{(1-\beta_i^4x^2)}{(1-\beta_i^2x^2)} - (1+a\beta_i^2x)Q(a,b,x) + (1-a\beta_i^2x)Q(-a,b,x)\right) \\ &\quad + \frac{\cos(\omega\Delta t)}{ax}\Bigg(\frac{(-1-a^2+b^2-2ax(a^2-b^2))}{1+ax}Q(a,b,x) + \\ &\quad\quad \frac{(1+a^2-b^2-2ax(a^2-b^2))}{1-ax}Q(-a,b,x)\Bigg) \\ &\quad + (1-\beta_f^2)((1+ax)Q(a,b,x)^3 + (1-ax)Q(-a,b,x)^3) \\ &\quad + (1+\beta_f^2)(Q(a,b,x)+Q(-a,b,x))\Bigg] \end{aligned} \tag{16}$$

with the abbreviation $Q(a,b,x) = \sqrt{(1+ax)^2 - b^2(1-x^2)}$.

The photon spectrum is given by the Poynting vector, i.e.

$$\begin{aligned} \frac{dI}{d\omega d\Omega_k} &= \frac{1}{2\pi} \mid r\vec{B}(\omega) \mid^2 \\ &= \frac{r^2k^2}{2\pi}\left|\sum_{lm}\left(a_m^{A(l)*}(\omega)\vec{O}_m^{V(l)}(k,\vec{r}) + a_m^{V(l)*}(\omega)\vec{O}_m^{A(l)}(k,\vec{r})\right)\right|^2 \end{aligned} \tag{17}$$

After changing the order of summation $\sum\limits_{lm} = \sum\limits_{l=0}^{\infty}\sum\limits_{m=-l}^{l} \rightarrow \sum\limits_{m=-\infty}^{\infty}\sum\limits_{l=0}^{|m|}$ and averaging over the azimuthal angle φ, i.e. averaging over all possible reaction planes, terms containing the product of two $\vec{O}_m^{V,A(l)}$ with different m drop out, which yields:

$$\frac{dI}{d\omega d\Omega_k} = \frac{r^2k^2}{2\pi}\sum_m\left|\sum_l ...\right|^2 \quad . \tag{18}$$

We will describe the spectrum with the nonvanishing terms of lowest order in l. In a similar way the photon emission was calculated for nuclear collisions below the Coulomb barrier [6].

The occuring products of the vector fields $\vec{O}_m^{A,V(l)}(k,\vec{r})$ determine the angular dependence of the spectrum. Due to the averaging over φ these products are real and depend only upon $\cos\theta_k$.
We use the far field limit for the functions $\vec{O}$:

$$\vec{O}_m^{A(l)}(k,\vec{r}) \;\approx\; \frac{e^{ikr}}{r}\vec{Y}_m^{(l,1)l}(\Omega_k)$$

$$\vec{O}^{V(l)}_m(k,\vec{r}) \;\approx\; -\frac{e^{ikr}}{r}\left\{\sqrt{\frac{l+1}{2l+1}}\vec{Y}^{(l-1,1)l}_m(\Omega_k)+\sqrt{\frac{l}{2l+1}}\vec{Y}^{(l+1,1)l}_m(\Omega_k)\right\} \tag{19}$$

Both, the product $\vec{O}^{V(l)*}\vec{O}^{V(l)}$ and $\vec{O}^{A(l)*}\vec{O}^{A(l)}$, respectively, yield the well known angular functions [3]

$$\begin{array}{ll}
\frac{3}{r^2 8\pi}\sin^2\theta_k & (l=1, m=0)\\[2ex]
\frac{3}{r^2 16\pi}\left(1+\cos^2\theta_k\right) & (l=1, m=\pm 1)\\[2ex]
\frac{15}{r^2 8\pi}\cos^2\theta_k\sin^2\theta_k & (l=2, m=0)\\[2ex]
\frac{5}{r^2 16\pi}\left(1-3\cos^2\theta_k+4\cos^4\theta_k\right) & (l=2, m=\pm 1)\\[2ex]
\frac{5}{r^2 16\pi}\left(1-\cos^4\theta_k\right) & (l=2, m=\pm 2)
\end{array} \tag{20}$$

For equal l the interference terms are asymmetric functions of $\cos\theta_k$. The evaluation yields for $l=2$

$$\begin{aligned}
&\vec{O}^{V(l)*}_m(|k|,\vec{r})\cdot\vec{O}^{A(l)}_m(|k|,\vec{r}) = \\
&\frac{(-)^{1+m}}{r^2 4\pi}\sqrt{\frac{5}{2}}\left(P_1(\cos\theta_k)\left(\begin{array}{cc|c} 2 & 2 & 1\\ m & -m & 0\end{array}\right)+2P_3(\cos\theta_k)\left(\begin{array}{cc|c} 2 & 2 & 3\\ m & -m & 0\end{array}\right)\right)
\end{aligned} \tag{21}$$

and for $l=1$ (only $m=\pm 1$)

$$\vec{O}^{V(l)*}_{\pm 1}(|k|,\vec{r})\cdot\vec{O}^{A(l)}_{\pm 1}(|k|,\vec{r}) = \mp\frac{3}{r^2 8\pi}P_1(\cos\theta_k) \tag{22}$$

For symmetric charge distributions, i.e. currents with even parity, the coefficients of these terms vanish.

The interference of dipole ($l=1$) and quadrupole ($l=2$) contributes with two different kinds of terms: both, the interference of magnetic and electric components of $\vec{B}$ and the interference of magnetic and magnetic as well as electric and electric components, respectively. The latter is asymmetric in $\cos\theta_k$ [6]

$$\begin{aligned}
&\vec{O}^{V(1)*}_m(|k|,\vec{r})\cdot\vec{O}^{A(2)}_m(|k|,\vec{r})\\
&=\vec{O}^{V(2)*}_m(|k|,\vec{r})\cdot\vec{O}^{A(1)}_m(|k|,\vec{r})\\
&=\frac{(-)^{1+m}}{r^2 4\pi}\sqrt{3}\left(\sqrt{\frac{3}{2}}P_1(\cos\theta_k)\left(\begin{array}{cc|c} 1 & 2 & 1\\ m & -m & 0\end{array}\right)+P_3(\cos\theta_k)\left(\begin{array}{cc|c} 1 & 2 & 3\\ m & -m & 0\end{array}\right)\right)
\end{aligned} \tag{23}$$

Hence, for a current with even parity the interference reduces to the product

$$a_{\pm 1}^{A(1)}(\omega)\vec{O}_{\pm 1}^{V(1)*}(|k|,\vec{r}) \cdot a_{\pm 1}^{V(2)*}(\omega)\vec{O}_{\pm 1}^{A(2)}(|k|,\vec{r}) = \pm\frac{1}{r^2 4\pi}\sqrt{\frac{15}{4}}P_2(\cos\theta_k) \tag{24}$$

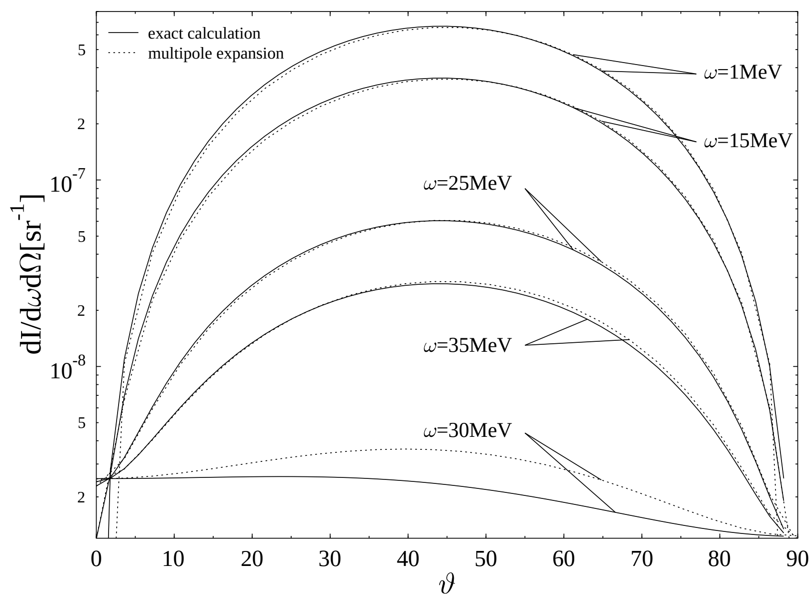

Figure 1: Comparision of the exact bremsstrahlung spectrum and the spectrum calculated by the first terms of the multipole expansion for a scattering angle $\theta_p = 3.6°$, the initial rapidity $y = 0.2$ and the final velocity $\beta_f = 0.5\beta_i$, for Δt we chose the large value of 20fm/c. The spectra are symmetric with respect to $\theta_k = \pi/2$. The transverse scattering generates finite radiation at $\theta_k = \pi/2$ even for soft photons ($l = 2$, $m = 2$ moment), the temporal phase between the two pairs of interaction causes the oscillation of the different terms in the expansion, i.e. the increase of the $l = 2$, $m = 1$ moment and therefore photon radiation in beam direction.

Here and in eq.(22) we used that we restricted the motion of the particles to a plane. The scalar product of $\vec{j}_m^{(l)}(\omega)$ with $\vec{e}_{\pm 1}^{(1)}$ in eq.(9) becomes a scalar product with $\vec{e}_x$ or $\vec{e}_y$. Choosing, without loss of generality, the $x-z$-plane as scattering plane, the coefficients $a_0^{A(l)}(\omega)$ vanish. For symmetric collisions the coefficients $a_m^{V(1)}(\omega)$ and $a_m^{A(2)}(\omega)$ vanish, too. The possibility to substitute $\vec{e}_{\pm 1}^{(1)}$ by $\sqrt{\frac{1}{2}}\vec{e}_x$ further simplifies the calculation since now complex conjugation of the coefficients $a_m^{A,V(l)}(\omega)$ and the switching of the sign of the index m decouple (see eq. (5)).

A comparision of the exact spectrum (16) and the first terms of the multipole expansion (18) in Figure 1 shows the quality of the restriction to terms of lowest order in l. This

restriction is valid as long as the initial and final rapidities can be described well enough by

$$y \approx \beta + \frac{\beta^3}{3} \tag{25}$$

For higher energies, terms of higher order in l have to be taken into account, i.e. the bremsstrahlung spectrum of ultrarelativistic particles is composed of infinitely many terms of the multipole expansion and their interference. Although the approximative character of this treatment is obvious, the multipole expansion offers a very instructive way to understand the behaviour of bremsstrahlung emission.

4 Comparision with data

To compare the spectrum obtained with the multipole expansion with experimental data, one needs to make further assumptions. We will perform an analysis of the radiation measured in the system $^{12}C+^{12}C$ at 84AMeV by Grosse et al [7] who determied the photon emission in minimum bias collisions. One therefore has to average the spectrum over all impact parameters b, weighted with the geometric cross section of the collision[1]:

$$\frac{d\sigma}{d\omega d\Omega} = \frac{1}{\omega}\int_0^{2\pi} d\varphi \int_0^{bmax} bdb \frac{dI}{d\omega d\Omega} \quad . \tag{26}$$

b_{max} can be chosen as $2R + x$, where R is the radius of the colliding nuclei and x is the surface thickness of the mass distribution.

We treat noncentral heavy ion collisions in the way proposed in [8], i.e. the charge $Z(b)$ involved in the collision is obtained from the geometric overlap integral [9]

$$V(b) = \int d^3x\, \theta(R^2 - x^2 - y^2 - z^2)\theta(R^2 - y^2 - (z-b)^2) \tag{27}$$

and hence $Z(b) = Z\frac{V(b)}{V}$, V and Z are the volume and the charge of the nucleus, respectively.

Earlier calculations exhibit the dipole structure[2] of the photon spectrum, assuming either the validity of incoherent summation due to an uncorrelated motion of the nucleons [10] or contributions from nucleus nucleus collisions at large impact parameters whose radiation should possess a dipole-like structure as well [11]. Even a mixture of coherent and incoherent radiation has been suggested for the description of the radiation [12]. In the following we want to propose as an alternative mechanism radiation produced by the correlated, collective motion of the nucleons. The current (13) is considered as a coherent elementary process in the heavy ion collision. For that reason we constructed the current (13) in a symmetric manner since the averaged final state of a symmetric heavy ion

[1] One assumes that, as it is valid for soft photons, the cross section of the radiation process factorizes into the cross section for the scattering process times the photon spectrum.

[2] According to the remark given in the following of eq.(25) the spectra of high-energy collisions should neither be called dipolar nor quadrupolar, unless these names refer to the first nonvanishing electric component of the expansion

collision is symmetric as well. Eq. (13) represents the simplest current which serves our purposes. We account for all processes which can be described by (13) considering one to four charged particles. For collisions involving neutral particles the corresponding terms in (13) are simply set equal to zero. We further assume that all nucleons collide at the origin, i.e. the current simply has to be multiplied by a temporal phase $e^{i\omega t}$ and integrated over time. The resulting spectrum of the nucleus nucleus collision is then similar to that of the nuclei feeling a box-like force $\sim \theta(T/2-\tau)\theta(\tau+T/2)$ in the time interval T, which here represents the time of the collision. This time dependence of the collision is also obtained, when one treats the collision of the two nuclei in the frame of a shock wave model, neglecting the spatial distance of the two outward travelling shock fronts, i.e. one performs the calculation as if the two shock fronts were sitting at rest in the origin (with this assumption, radiation is calculated in [13]).
Adopting the shock wave picture, we can estimate the minimal collision time with a shock velocity β_{sh} equal to the speed of light,

$$T_{min} = \frac{D}{\gamma_i(\beta_i + 1)} \quad , \tag{28}$$

D is the diameter of the nuclei. For noncentral collisions the time is taken to depend linearly on the diameter at $b/2$.

To obtain the correct behaviour of the spectrum with respect to the photon energy, one can introduce a phase space correction [14]

$$\frac{R_2(\tilde{s})}{R_2(s)} = \frac{\lambda^{1/2}(\tilde{s}, m_1^2, m_2^2)}{\lambda^{1/2}(s, m_1^2, m_2^2)} \frac{s}{\tilde{s}} \tag{29}$$

with $\tilde{s} = s - 2\omega\sqrt{s}$ and $\lambda(x, y, z) = (x-y-z)^2 - 4yz$ the kinematical triangle function; m_1 and m_2 are the masses of the colliding particles, respectively. This way of treating the spectrum of high-energy photons is strictly justified only for the incoherent summation of the spectrum. Since no adequate formalism is available at the moment, however, we adopt this correction factor (29) also for coherent emission, i.e. we multiply the resulting spectrum by (29). To describe the high-energy tail of the spectrum it is unavoidable to consider either three- and four-nucleon clusters which feed their energy to the photon production in a collective manner, as proposed in [15]. We increase the nucleon momentum by the mean Fermi-momentum which has to be taken into account since it is of the same order of magnitude in the considered system. Consequently, the available phase space of the photons gets enlarged.
In additionally to the decreasing available phase-space, the time dependence of the collision determines the emission of high energy photons, too. Since the energy distribution of the spectrum is the square of the Fourier-transform of the acting force, the spectrum of the considered process is poportional to

$$\frac{dI}{d\omega d\Omega} \propto \frac{1}{\omega^2 T^2} \sin^2(\omega T) \tag{30}$$

Since $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ and both, ω and T enter symmetrically in the spectrum, one obtains the soft photon limes for either $\omega \to 0$ or $T \to 0$ (for all photon energies). Therefore, for short collision times one obtains a nearly constant, flat spectrum. In this way, the collision time controlls the slope of the spectrum for not too large photon energies. A lower bound for T, however, is given by (28) and has the approximate value of 5fm/c.

As mentioned earlier, the presence of asymmetric collisions can be expected,which are the consequence of fluctuations of the charge distribution in the nuclei. The probability for these asymmetries of the charge distribution in the overlapping volume as a function of the impact parameter we parametrize in the following way: For nearly central collisions it is zero (the probability for a symmetric collision is 1) and smoothly drops to 1/2 when the mean charge of one nucleus in the overlapping volume becomes ≤ 1. The probability for symmetric collisions approaches 1/4, the remaining fourth is the probability for symmetric but radiationless collisions, i.e. neutron neutron collisions (see Figure 2).

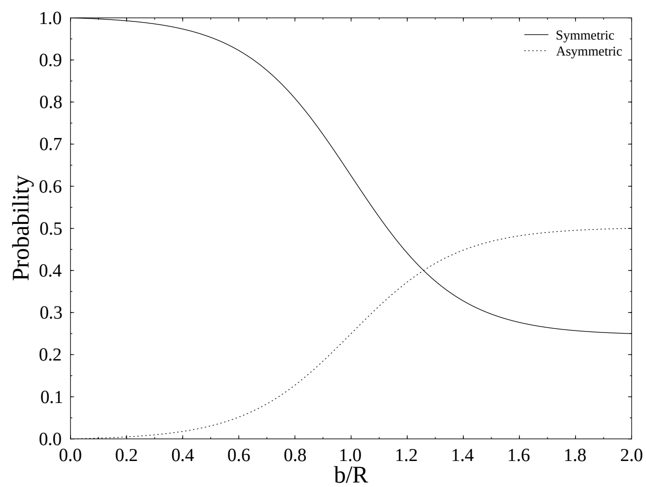

Figure 2: Parametrization of the probability for asymmetric and symmetric collisions as a function of the impact parameter. For convenience, the functions used here are for both cases a hyperbolic tangent. For symmetric collisions: $P_{symm} = \frac{1}{a}\frac{3}{8}\tanh(\mathrm{artanh}(a)\cdot((b/R)-1)) + \frac{5}{8}$ and for asymmetric collisions: $P_{asymm} = \frac{1}{a}\frac{1}{4}\tanh(\mathrm{artanh}(a)\cdot((b/R)-1)) + \frac{1}{4}$. a determines the stiffness of the parametrization and is taken to be $a = 0.99$.

As possible asymmetric and symmetric collisions we consider all possible combinations of up to four charges in (13) with one exception: instead of the collision of only two charged particles (one from the projectile and the target) possessing a temporal distance Δt between the scattering processes, we consider the symmetric case with two particles at each case, since it is unlikely that all particles scatter in only one transverse direction. The nonvanishing coefficients are, besides a factor two which is canceled since only half of the charge enters in the latter case in the calculation, in both cases the same. In the

latter however only coefficients with even m exist.
The "delay"-parameter Δt breaks possibly present symmetries of the scattering process. In this way Δt controlls (with respect to ω) the strength of the contribution of different terms of the expansion. Processes with a symmetric charge distribution in the overlapping volume of the colliding nuclei can therefore (for certain photon energies) generate a purely dipolar structure in the photon emission. This observation is similar to the result in [11]. The asymmetric collisions generate also the interference terms of types (21)-(23). However, since the probabilities of the asymmetries are invariant under a parity transformation, these terms cancel when averaging over all possible asymmetries.
A quantitative evaluation shows that the influences of different parameters counterbalance to a certain extent. These are e.g. the collision time and the cluster size for higher photon energies and Δt (for intermediate photon energies) or the strength of the stopping (for soft photons), respectively, and the ratio of symmetric to asymmetric collisions.

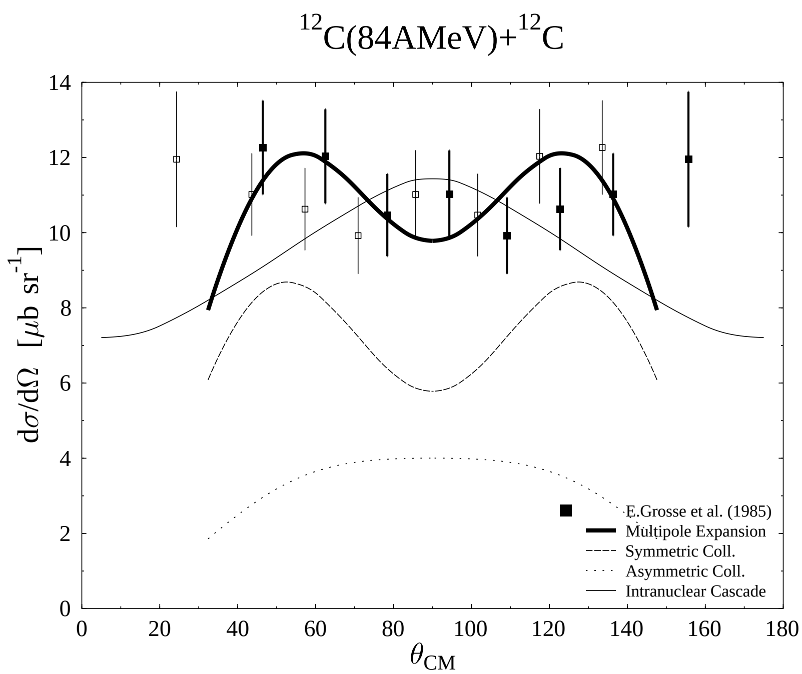

Figure 3: The bold line shows the spectrum obtained with the dipole ($l = 1$) and quadrupole ($l = 2$) terms and their interference. Contributions of collisions with an assumed asymmetric or symmetric charge distribution in the nuclear overlap region are shown as well. The angular distribution of the data [7] (symmetrized with respect to $\theta = 90°$, open squares) can be described quite well. The thin line represents the result of an intranuclear cascade model [10]. The cross section shown here is integrated over ω between 50 and 100 MeV.

Figure 3 shows the spectrum obtained with the discussed assumptions (full line). The

dashed line represents the spectrum calculated in the frame of the intranuclear cascade model [10]. The angular distribution is not of dipole type and represents the interplay of the different multipole components and their mutual interference. Confronting the data with the calculation, the measurement seems to indicate an even stronger contribution of the quadrupole. One may expect that the photon angular distribution gets modified when a larger number of interacting particles is included in the source current to allow for other asymmetries with a different ratio of the $l = 1$ and $l = 2$ terms. With better knowledge of the charge fluctiations one can improve the assumed impact parameter dependent ratio of asymmetric and symmetric collisions. E.g., on has to estimate the influence of the repulsive Coulomb forces on the protons in the nuclei.

One possible signal of the slope of the energy spectrum in Figure 4 is, that the time of the collision is much shorter than one might infer from the naive estimation, simply regarding the time the nuclei need to pass the distance D/γ_i, which gives approximately 25fm/c for the considered process. Figure 4 shows an arithmetically averaged spectrum for collision times between 6 and 16fm/c. The gap at soft photons could be closed by adjusting the ratio of symmetric to asymmetric collisions.

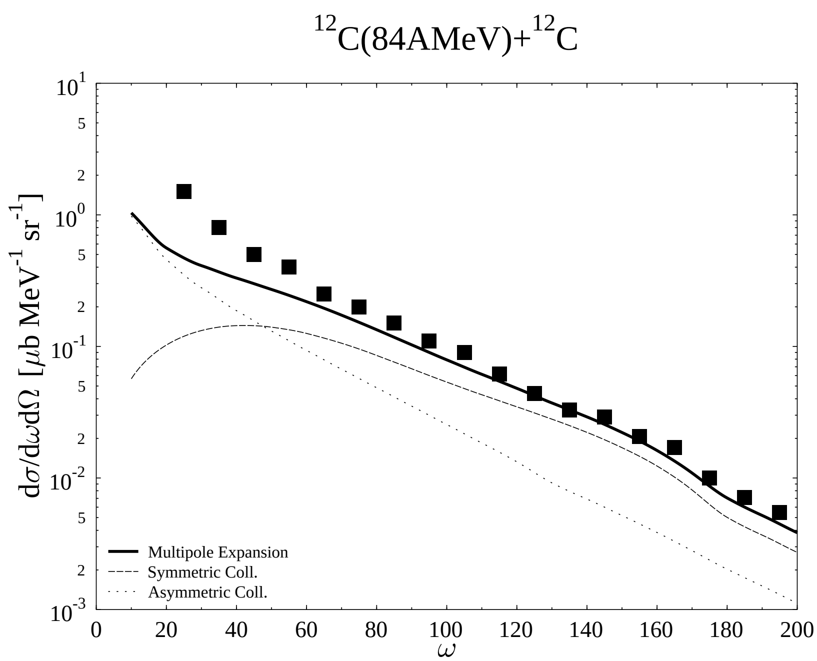

Figure 4: Comparision of the measured energy distribution at $\theta_k = 90°$ with the result of the multipole expansion.

According to Fig. 4 and the considerations made above, the angular distribution should change with photon energy. To produce a photon with high energy, one needs

many particles which deposit their energy in one photon. Since the probability to form large clusters depends on the number of involved particles, collisions with small impact parameters, i.e. more symmetric collisions, contribute more to the high energy tail of the spectrum.

For the calculation we used the following values for the parameters: For the mean total momentum we assumed 320 AMeV. The final velocity was taken to be 8% of the initial velocity, Δt is approximately one third of the actual, b dependent collision time. Since the final velocity is very small, the effects of the transverse scattering are small as well. The scattering angle was choosen to be $14°$. These values gave the observed agreement with the data.

5 Conclusion

The multipole expansion of electromagnetic radiation as already presented in [4] applied to bremsstrahlung provides a transparent understanding of the radiated fields. Compared to the full calculation the isolated handling of the multipole components and their mutual interference allows for a deeper insight in the influence of different parameters and properties of the current as, e.g., transverse scattering and the presence of symmetries.

The fair agreement of the calculation presented in this paper with the data is no proof for the correctness of the model. It demonstrates, however, that a completely coherent treatment of bremsstrahlung is able to describe the data. The "exponential" shape of the energy spectrum, which was interpreted as being caused by incoherent photon emission from the uncorrelated motion of particles in a thermalized nucleon gas [16] can alternatively be described by the coherent photon emission of the strongly correlated motion during the stopping phase in the collision. Our simple model assumptions have to be supported by an extended calculation, where a detailed stopping mechanism, as e.g. in [12], is assumed and the spatial extension of the colliding particles is taken into account.

We conclude, that the measured data, especially the integrated cross section, do neither show information about thermalization, though the angular distribution was even interpreted as representing isotropic radiation [12], nor provide a proof for incoherent photon production.

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